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Math Acceleration

This is some advice on taking advanced math courses. What I've written is best used as a guide, not a prescription. Read the parts that seem most relevant to you.

Background Information

The default math path is Prealgebra in eighth grade, then Algebra I, Geometry, Algebra II, and Precalculus in the following years. However, there are two main courses for acceleration. In fifth grade, with teacher approval you can take a test to qualify for Prealgebra the next year in sixth grade. Everyone in sixth grade (excepting those in Prealgebra) takes a test to determine whether you take Prealgebra or Math 7 the next year. However, this test is being discontinued, and in addition the ICCSD district is shifting to a middle school model with 6th, 7th and 8th graders together, which will further complicate math advancement tests.

Importantly though, the district provides yearly tests in August before school starts where you can advance one grade of math. For example, if you're on track to take Algebra II but have already studied all its content, you'll be able to take Precalculus instead by passing the test.

Why Skip?

Being in a class that can't interest you is a tragic waste of time. The reason why the math progression is so slow is because memorizing things you don't care about is the most tedious process to endure. That's the path most people will take through math. But for any bright student, their sixth grade math class is unbearable.

Fortunately, most of these problems can be alleviated by taking accelerated classes. The more mathematics you learn, the more you appreciate its beauty. There is so much meaning hidden in the humble circle and quadratic equation, but it's hard to find in the bland pages of your school prealgebra textbook. Taking advanced classes is your first step to grasping these concepts.

When you do skip, you'll most likely be placed in a class with people a few years older. As a fairly extreme example, in seventh grade I was taking precalculus with tenth graders. While it can be intimidating to make the trek to your local high school (or junior high if you're in elementary), you'll have a rare opportunity to get familiar with secondary school. Take it to its fullest.

Planning

Most people reading this will be from sixth to eighth grade, and at that age you still have enough years of school left to seriously consider how far you intend to skip. There are two relevant questions: when do you plan to take calculus, and what math courses have you already taken? There are a lot of flaws with this approach, but it is useful as an illustration.

On the standard path, you won't take calculus until college. Being 1x accelerated means it'll be in 12th grade, 2x accelerated in 11th. A considerable number of people are 1x accelerated, but further advancement is very diminished. Only a few people are 3x accelerated, even less are 4x, and I know only two people who took calculus in eighth grade.

You can draw your own conclusions from those numbers, but what I've seen is that most people never bother to go further along with skipping math courses after they take the 5th or 6th grade test. There's a tremendous amount of potential that isn't getting realized.

It's perfectly fine to be content with being 2x accelerated. What you shouldn't do is be 2x accelerated, find your classes to be boring, but think that you can't skip further. Don't be afraid.

What about the opposite scenario? How can you tell if you're being too ambitious? The important thing to note is that you probably don't know what you're capable of. When I was in elementary school, I certainly didn't envision taking Calc BC in eighth grade. I think what's really important is to recognize the amount of effort it requires to achieve something, and to be able to evaluate if you are able to put in that amount of work. If you can do those two things, you never have to worry about being too ambitious.

Finally, it's perfectly fine to not know how far you want to go in math. Life is confusing, and it's unfortunate that you get benefits by making hasty decisions instead of decisions that are better in the long run. Importantly, it's never too late. It is always better to be in a class that excites you than to be in a class that doesn't.

Recommendations

What follows are some of my recommendations for taking various courses in various grades. When it says "7th grade", for example, that means that you will be entering 7th grade when fall comes. If you go down from there, when it says "Taking Algebra I", that means that this is a scenario where you would be taking Algebra I in 7th grade. If I say I have no recommendation against taking a course, that just means that I don't see any problem with taking that course if you feel like it suits you.

If you are entering

6th grade: There are only two scenarios: taking Prealgebra or taking Math 6.

Taking Prealgebra: No recommendation against.

Taking Algebra I: No recommendation against.

Taking Geometry: This does mean that you'll have to take two tests. It's not that much of a hassle, if the teacher administering the tests is permissive.

Taking Algebra II: At this point, you would have to go to the local high school. The jump from middle school to high school isn't prominent but it is important to the rigor of the course. This also means that you'd have to do some pretty intense wrangling with whoever administers the tests.

7th grade:

Taking Math 7: This is one of the few courses I would definitely advise against. The new material in this class is pretty scant, and it functions mostly as a year-long review for Prealgebra.
Taking Prealgebra: Instead of Math 7, this is what I'd recommend trying to take.
Taking Algebra I: No recommendation against.
Taking Geometry: Fine if you took Algebra I or Prealgebra the year before. I haven't heard of anyone who was going to take Math 7 but skipped twice to take Geometry, but I see no reason why it couldn't be done.
Taking Algebra II: I'd caution against taking this course especially if you want to go directly to Calculus after this, as the test to skip precalculus is notorious for being one of the hardest.
Taking Precalculus: The most likely way you'd get here is by taking Geometry the year before and then skipping Algebra II.

8th grade:

Taking Prealgebra: I would advise against taking this course in 8th grade; you'll most likely be wading through tedium.
Taking Algebra I: I'd highly recommend taking this instead of Prealgebra. If you took Prealgebra the year before, I'd be personally inclined to skip, but it's up to you.
Taking Geometry: No recommendation against. I haven't heard of anyone skipping from Prealgebra to Geometry, but I don't see why it couldn't be done.
Taking Algebra II: Wouldn't recommend it if you want to take Calc in 9th grade. In addition, I wouldn't recommend trying to skip Geometry. This class is pretty much the only time you'll ever learn geometry at school, and it's useful to start with a solid foundation.
Taking Precalculus: No recommendation against.
Taking AP Calculus AB/BC: At last, we've arrived at Calculus. I would definitely advise against taking AB. AB basically stretches out your first semester of calculus in college into a whole year. But BC is two semesters of calculus, which is what you'd do anyway, and you'll also earn the AP credit for AB. If you've made it this far ahead, you're definitely able to handle BC.

9th grade:

Taking Algebra I: Would advise against. I do want to talk about being in the default math path in high school though. My main point is that it's never too late to skip. Don't consign yourself to four years of boredom because you think it's impossible for anything else to happen. People often look down on others who are on the standard path, but they're wrong. Not being 1x or 2x accelerated doesn't mean you're less intelligent. All people are capable of understanding advanced math; they're just on different timelines. Skip math classes because staying wouldn't benefit you, not to prove that you're intelligent.

Taking Geometry: No recommendation against.
Taking Algebra II: No recommendation against, although once again I'd caution against skipping Geometry.
Taking Precalculus: No recommendation against.
Taking Calculus: Once again, I heavily recommend BC.
Beyond: If you did AP Calc last year, congrats! You've finished all the math that the district ever anticipated you would learn, and quite a bit faster. But now you have more opportunities at the university. The important thing to note is that registration for college courses in the fall takes place in March. You'll have to contact a high school counselor while still in 8th grade. Make sure you fully understand and go through the entire registration process; the deadlines to register are unnegotiable.

If you're going to be in 10th grade or above and you're reading this, nothing really changes about my advice. My thoughts are mostly about the classes themselves, not what grade level you should take them at.

How to Study
Skipping math courses is one thing, but studying for it is another.

When Should I Start?
The short answer: As soon as possible without disturbing anything else you'd like to do (homework, extracurriculars).

Because of this, it's common for students to start during summer break. It's surprising that all of the content of a nine-month course can be learned effectively in just three months, but the environment at home can be much more productive than the environment at school. I would say that starting twelve weeks before is probably the comfortable limit. While I can conceive of someone studying and passing their test in six weeks, you would have to give up pretty much anything else you wanted to do over the summer.

It's useful to think of time commitment in terms of total hours you'll need to expend. I would estimate that the number of hours for any math course would be between 60-120. It's important to note that the number will necessarily be different for different people, and it also varies by course.

From our estimate above, if you were to start in the summer, you'd probably be looking at a little bit more than an hour a day of study. One hour is both a small and large number. It's small in the sense that you have 24 hours in a day, so it's not that demanding. But it also means one hour straight wrestling with a textbook full of new material. If you're new to studying, this is a formidable commitment.

Studying is most effective when you have a set time every day for you to focus just on math. Not having any flexibility is an expensive requirement, and chances are you won't be able

to have the same study time every single day. But having a regular schedule makes it easier for your mind to build a routine. Getting good at math is mostly about doing a lot of math. Integrating it into your daily life makes it a lot easier.

What Do I Use?

My recommendation would be a physical copy of one of AoPS's books. However, other textbooks you happen to have already are fine.

The textbook industry tries to make profit, not to make good books. Of course, poorly written books aren't profitable, but what you'll get from Amazon or other stores is just a sea of mediocrity. For this reason, I'd recommend against trying to buy a textbook from Holt, McDougal, or one of the numerous other titles out there; while the book is being used for an average high school course, for self-studying the cost isn't worth it. If you happen to have a textbook lying around at home already, feel free to use it, but don't assume that it's high quality.

However, there is one series of books I believe to be some of the best available. These are Art of Problem Solving's curriculum. It covers everything in the school math sequence from Prealgebra to Calculus. Art of Problem Solving has been making educational materials for some of the best students in math competitions for a long time, and many of the same books you can use to study for math acceleration are being used as training for MATHCOUNTS, AMC and AIME. While the books are relatively expensive, I personally believe their cost is worth it.

I do have a warning: these books can be *very* hard, and cover material that will likely not be covered in your typical high school class. This is most prevalent with the Intermediate Algebra and Precalculus textbooks; both include problems that come from some of the hardest math competitions available. If you are studying with one of these, I definitely would recommend starting much earlier, as properly working through them can take many months.

But what if you don't want a physical book? I have mixed feelings about online textbooks. As much as online materials are hyped for being portable and instantly available, I find the experience of being able to physically turn pages and read a warmly lit paper page a lot better than fumbling with Pearson's online eReader. But this is entirely a matter of personal preference.

Now let's look at online math resources you can find. The one you've probably heard of is Khan Academy. Khan has a solid math curriculum, but I would caution against solely using it to study. The way things work, you'll learn how to solve problems, but only in narrow contexts. There's very little creative thinking involved, and solving hard problems requires creativity. It's good as a source of easy problems and for learning your basics, but fails to build serious problem solving skills. This doesn't mean you shouldn't use it, but it's best used as a complement to a textbook.

The other math resource I want to mention is AoPS's Alcumus training tool. All it requires is creating an AoPS account, and you have access to thousands of problems, most of them from reputable math competitions or created by the AoPS staff themselves. It's quite a good

complement to studying with AoPS textbooks, and it's a good source of innovative and difficult problems.

There are other math websites available, but I don't know many specifics about them. If there's a resource you prefer, feel free to use it. But the way to learn math is by doing math. Your priority should be solving problems, and then solving harder problems.

Studying Tips

The best way to learn math is to do math problems. Any studying that doesn't involve that is ineffective.

First, some general tips that apply for any studying you'll do: don't have distractions. Focus on math and math only when you're studying. Some people think that music helps them study better, and no one's stopping you from listening, but personally I find it too much of a distraction (especially when you get an Spotify ad every fifteen minutes). If you're in middle school, by now you should have found out that trying to work on two assignments at the same time doesn't work, and neither does trying to study for two different things, so put away your English homework. In addition, try to avoid using Youtube, social media and your other favorite sites. Particularly for social media, nothing anyone will say is important enough for you to check every five minutes for new messages and stop paying attention to math.

Learning math should feel natural, with periods of frustration and inspiration. What it shouldn't feel like is a constant slog through material you have no interest in. You should be frustrated because a problem is more difficult than anything you've faced before, not because you have to solve fifty problems with the exact same tedious technique.

It's pretty reasonable for most of your time to be spent with the textbook, but don't dedicate your entire time to it; especially if you're not using AoPS, you could be missing out on some material. I'd say that something like spending your weekdays working on the textbook, and weekends with Alcumus or Khan Academy is pretty ideal.

Importantly, different strategies work for different people, so if some of what I've proposed doesn't work for you, don't continue doing it; find another strategy.

Before the Test

I would say an ideal ratio of time spent learning new material to time spent reviewing is probably 3:1, or somewhere upwards of that. The main goal of review is to make sure you have a good understanding of previous concepts. This means that it should be fairly quick; you either know it well or you don't, and if you don't, you do more review.

If you're trying to work on a weak area, do problems specifically about that topic. Alcumus and Khan Academy work well for this. In addition, run through a comprehensive assessment of the class. Unfortunately the school district doesn't publicly release old exams, but ones you find online or in textbooks are sufficient. I really wouldn't go through an exam more than three times at most, and the point of taking a practice assessment is to assess how prepared you are for the real thing, so don't use them as your sole review.

Next, what should you be doing in the three days leading up to the test? Well, my recommendation is sleeping well, eating well, and hanging out with friends. There's really nothing that you could do a few days before the test that would significantly benefit you. The best way to relax and feel confident is to take a break from studying, not to study more.

Your parents will probably want you to continue reviewing up to the day before. I don't think that it'll make you worse prepared for the test, but it will be mostly pointless. As a final caution, of all the self-destructive things you could do, the worst is staying up late and cramming.

During the Test

You've just been handed your test paper, so now what? I don't really think that there's one superior test strategy that everyone should use for success. But there are two things you want to avoid: running out of time and doing problems wrong. The issue is that trying to be more accurate takes more time.

The questions you'll be given don't require much ingenuity. In most cases, it comes down to how well you know the content. This means that most of the time pressure is from questions that will naturally take more time than others. So my recommendation is to go through the test and look for questions that you can easily solve, and do those. Once you've reached the end, go back and now attempt the problems that are easy to solve but time consuming. Only then should you consider trying to do questions that you don't see an immediate solution for.

What about accuracy? Firstly, read questions carefully. The main objective is to make sure you don't miss out on easy points, and your strategies for doing this will differ. Some teachers will tell you to underline key parts of a problem, but that's always seemed condescending to me; my understanding isn't magically improved because there's a line underneath a word I recognize. But read carefully.

Second, check again. In particular, make it easier for yourself to check your work; misreading your 4s as 9s and other handwriting issues have led to woe. When you check, you're verifying that your logic is correct, but also that you put in the right numbers. If either of those go awry, that's a hit to your score.

The way I do tests is usually to go in rounds of solving new problems and checking previous answers. If you're running short on time after finishing a round, I would say you should check your answers over trying to solve new problems, unless you think your current score isn't enough to pass, or if you're already close to answering a question but need some extra work.

Remember that in the end your success mostly depends on how well you prepared. If you've ingrained all the material, your testing strategy probably won't make a difference. Similarly, if you don't study, no matter how good your strategy is, you won't be able to solve enough problems to pass.

Aftermath

Hopefully you'll find success. But what happens when the email arrives and you find out your score was below the threshold?

Maybe you could have spent a bit more time reviewing matrices or started earlier, but that's useless to contemplate. The present you has no control over the past you. But the test you've been studying for is also flawed. One score can't represent your knowledge of math! When administrators develop the difficulty of the exam and decide what score passes, they're trying to balance next year's statistics. They want to find"qualified" students, but no test can ever do this perfectly. Regretfully, the system will fail people who could have thrived in an advanced math class.

But it doesn't matter why you didn't pass; all that matters is that a whole summer of studying is wasted. Or has it been?

One of the especially tragic realities is that you've been studying essentially to make the actual class a waste of time. There is nothing you can do to unlearn precalculus. But new opportunities have emerged. People can learn a whole year's worth of content in three months. It is perfectly possible for you to study Algebra I while you're consigned to Prealgebra. With a significantly longer time period to study, you'll have a much more certain chance of passing next time.

If you're stuck taking Algebra II while knowing all the content, take advantage of it. Homework will be easy to complete; so will tests. During class you can instead study whatever you like. Free time is precious during the school year; now you have a whole school period.

The most worrying thing you'll hear is that failing to pass has ramifications for college. Firstly, if you're in middle school, it's far too early to be seriously worrying about college. Secondly, it really has little effect on your chances of getting in. There are a lot more important things to colleges than what year you'll end up taking calculus. Prestigious universities like MIT and Harvard have freshman year calc classes, just like every other college in the nation. In no way is being accelerated in math a hard requirement.

The best way to show colleges that you really care about math is not through courses; instead, it's in the world of competition math. I've written more about that elsewhere. My main point is that one score on a test does not dictate your future life.