Quadratics Handout

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1   Problems

1. The graph of $y = ax^2 + bx + c$ is a parabola with vertical axis of symmetry. The vertex of this parabola is $(2, 3)$ and the parabola contains the point $(4, 4)$. Find the value of y when $x = 6$. (Alcumus)

2. Let d and e denote the solutions of $2x^2 + 3x - 5 = 0$. What is the value of $(d - 1)(e - 1)$? (2003 AMC 10A #5)

3. A parabola with equation $y = x^2 + bx + c$ passes through the points $(2, 3)$ and $(4, 3)$. What is c? (2006 AMC 10A #8)

4. The quadratic equation $x^2 + mx + n$ has roots twice those of $x^2 + px + m$, and none of m, n, and p is zero. What is the value of n/p? (2005 AMC 12B #12)

5. There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x. What is the sum of those values of a? (2005 AMC 10A #10)

6. For how many values of a is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$? (2005 AMC 12B #8)

7. George has a quadratic of the form $x^2 + bx + \frac{1}{3}$, where b is a specific negative number. Using his knowledge of how to complete the square, George is able to rewrite this quadratic in the form $(x + m)^2 + \frac{1}{12}$. What is b? (Alcumus)

8. The line $y = 3$ intersects the graph of $y = 4x^2 + x - 1$ at the points A and B. The distance between A and B can be written as $\frac{\sqrt{m}}{n}$, where m and n are positive integers that do not share any factors other than one. Find the value of $m - n$. (Alcumus)

9. Points A and B are on parabola $y = 3x^2 - 5x - 3$, and the origin is the midpoint of $\overline{AB}$. Find the square of the length of $\overline{AB}$. (Alcumus)

10. Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that

$a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of the equation $x^2 - px + q = 0$. What is q? (2006 AMC 10B #14)

11. The parabola $y = ax^2 + bx + c$ has vertex (p, p) and y-intercept $(0, -p)$, where $p \neq 0$. What is b? (2006 AMC 12B #12)

12. Find all numbers a for which the graph of $y = x^2 + a$ and the graph of $y = ax$ intersect. Express your answer in interval notation. (Alcumus)

13. A parabola with equation $y = ax^2 + bx + c$ is reflected about the x-axis. The parabola and its reflection are translated horizontally five units in opposite directions to become the graphs of $y = f(x)$ and $y = g(x)$, respectively. Which of the following describes the graph of $y = (f + g)(x)$? (2003 AMC 12A #19)

14. The parabola with equation $p(x) = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $q(x) = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$? (2001 AMC 12 #13)

15. Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$? (2011 AMC 12A #14)