Iowa City Math Tournament Countdown Round

Iowa City Math Circle

August 9, 2020

Rules

- The top 8 participants after the individuals rounds will be able to participate in the Countdown round.
- Each round consists of two players.
- There will be a total of 7 rounds.
- In the first 4 rounds, the winner will be the participant who gets the most correct questions out of 3 questions. If participants are tied after 3 questions, the first person to get the next question correct advances.
- In the last 3 rounds, the winner will be the participant who gets to 3 correct questions first.
- Participants should have their mics unmuted during their rounds.

Rules (Continued)

- Participants have 45 seconds to answer each question. To answer, the participant must first type x in the Zoom chat. Once acknowledged by a moderator, the participant should verbally say their answer within 3 seconds.
- If a participant gives a wrong answer, they cannot answer again.
- If a participant answers incorrectly, the other participant will have the remaining time to answer, unless they've already answered.
- Spectators who interfere with the competition by revealing answers on the zoom chat or causing other disruptions will be disqualified.

Question 1

What is $(1 + (1 - (1 \times (1 \div 1))))$?

Ans: 1

Question 2

Kobe is practicing free throws before his basketball game. If he has made seven out of his first ten free throws, how many throws does he have to make over the next five free throws in order to get his free throw percentage over 75%?

Ans: 5 free throws

Question 3

How many ways can a club leader choose two of the 10 club participants to form a committee?

Ans: 45 ways

Question 4

If a cube has surface area 216, what is its volume?

Ans: 216

Question 5

Evaluate the product $2020 \cdot 2019 \cdot 2018 \cdots 2 \cdot 1 \cdot 0$.

Ans: 0

Question 6

A list of number starts off with 2 and 5. After the second number, each number in the list is calculated by multiplying the previous two numbers (so the third number is 10). What is the 6th number?

Ans: $25,000$

Question 7

A specific heptagon has an area of 65. If each the side lengths of this heptagon is doubled, but with each angle staying constant, what is the area of the new heptagon?

Ans: 260

Question 8

A fair coin is tossed 4 times. What is the probability that an even amount of heads will occur out of these 4 tosses?

Ans: $\frac{1}{2}$

Question 9

Find the area of an isosceles trapezoid if the base lengths are 1 and 2 and one of the interior angles measures 60 degrees.

Ans: $\frac{3\sqrt{3}}{4}$

Question 10

One day in Iowa, the air temperature recorded was -35° F. Three days later, the air temperature recorded was 37° F. By how many degrees did the air temperature increase per hour during this three day period?

Ans: 1

Question 11

Find the closest integer to the solution x in $x^3 - 3x^2 + 3x - 1 = 7999$.

Ans: 21

Question 12

The seven books in the Harry Potter series have page lengths 607, 233, 636, 251, 766, 607, and 317. What is the difference between the median and mode of the page counts?

Ans: 0 pages

Question 13

John has 60 marbles that he wants to put in some bags. If he wants to have each bag containing a different amount of marbles, what is the maximum amount of bags that he can use?

Ans: 10 bags (Note: 60 is also accepted because a bag can be placed inside a bag)

Question 14

A line is drawn from $(0, 0)$ to $(16, 20)$. Including the endpoints, how many lattice points does the line go through? A lattice point is any point in the coordinate plane whose coordinates are both integers.

Ans: 5

Question 15

The product of 189 and x is a perfect cube. What is the smallest possible value of x, if x is a positive integer?

Ans: 49

Question 16

A 3 by 3 grid containing integers exist. The number at the top left corner of the grid is 1. Additionally, each integer on this grid is 1 more than the integer directly left of it, and 1 less than the integer directly above it. What is the sum of all the integers in this grid?

Ans: 9

Question 17

How many 5 digit numbers can be created if every single digit must be even? The number can not start with a 0.

Ans: 2500

Question 18

Compute the remainder when 47^3 is divided by 7.

Ans: 6

Question 19

Find the value of (((((2!)!)!)!)!)!.

Ans: 2

Question 20

What is the area of a rectangle whose perimeter is 20 and has all of its side lengths composite numbers?

Ans: 24

Question 21

Find the sum of the roots of $x^2 + 10x - 4\pi$.

Ans: -10

Question 22

Find the sum of the distinct prime factors of 2020.

Ans: 108

Question 23

6 people, including Kevin and James, are randomly seated at a circular table that has 6 chairs. What is the probability that Kevin is not seated adjacently to James?

Ans: $\frac{3}{5}$

Question 24

How many ways are there to arrange the letter sequence "CORONA" if the first letter must be a consonant?

Ans: 180 ways

Question 25

The angles of a triangle form an arithmetic sequence. If the smallest angle is 1, what is the largest angle?

Ans: 119°

Question 26

A rectangular prism has dimensions l, w, and h. If $l + w + h = 10$, and if the surface area of the prism is 40, what is $l^2 + w^2 + h^2$?

Ans: 60

Question 27

A number is randomly chosen from 1 to 100 inclusive. What is the probability that the number is relatively prime to 10?

Ans: $\frac{2}{5}$

Question 28

There is a tile grid with dimensions n by m, such that there are 30 tiles that do not touch the edge of the grid. What is the smallest possible value of nm?

Ans: 56

Question 29

A chess knight can move to any lattice point $\sqrt{5}$ units away in each move. For example, a knight at $(0, 0)$ can move to $(1, 2)$, but not $(1, 1)$ or $(2, 0)$, in one move. What is the minimum number of moves a knight has to take to get from $(1, 1)$ to $(7, 7)$?

Ans: 4 moves

Question 30

The Cedar Falls golf team participated in a golf meet with 18 other teams. If the number of teams they scored better than is double the number of teams they scored worse than, what place did they get? Assume there are no ties.

Ans: 7th place

Question 31

Find the maximum area of a triangle that has side lengths $6, 6$, and x.

Ans: 18

Question 32

How many equilateral triangles with side length 1 inch are needed to tile an equilateral triangles with side length 6 inches (with no overlap)?

Ans: 36 triangles

Question 33

In trapezoid $ICMT$, IC is parallel to MT. If the distance between IC and MT is 20 and the area of $ICMT$ is 2020, find $IC + MT$.

Ans: 202

Question 34

The integers 1 through 5 are written on slips of paper and dropped into a bag. Jacob draws two slips with replacement. What is the probability that sum of the two numbers drawn is odd?

Ans: $\frac{12}{25}$

Question 35

Find the area of a hexagon with side length $\sqrt{2}$.

Ans: $3\sqrt{3}$

Question 36

Jim has 40 of cards, 10 being each of blue, red, green, and yellow (a card cannot have multiple colors). Find the probability that when Jim randomly draws two cards without replacement from the deck, he ends up with two cards of different color.

Ans: $\frac{10}{13}$

Question 37

Sam has four rods, with lengths 1, 2, 3, and 4 units. How many distinct triangles can he form out those rods?

Ans: 1

Question 38

Find the smallest two-digit positive integer such that its digits are both multiples of 4. A number here can not start with the digit 0.

Ans: 40

Question 39

Evaluate $1 - 3 + 5 - 7 + 9 - 11 + \cdots + 97 - 99$.

Ans: -50

Question 40

Pens come in packs of 5 and in packs of 6. If customers can only buy complete packs of pens, what is the greatest number of pens that cannot be bought?

Ans: 19 pens

Question 41

Bob builds a chicken pen with 20π feet of fencing. What is the greatest possible area of the chicken pen?

Ans: 100π

Question 42

Point B is located at $(0, 0)$, point R is located at $(0, 2)$, point U is located at $(5, 5)$, and point H is located at $(3, 0)$. Find the area of quadrilateral $BRUH$.

Ans: 12.5

Question 43

A boy learns Japanese at the rate of 40 words per hour. Assuming this rate stays constant (yes pretend he does not sleep), how many hours will it take him to learn the entire Japanese dictionary, which contains $100,000$ words?

Ans: $2,500$ hours

Question 44

Richard's investment in AoPSCoin grows exponentially at a rate of 10% per day. Richard has 100 AoPSCoin on Monday, which grows to 110 AoPSCoin on Tuesday, and so on. On what day of the week will he have over 160 AoPSCoin?

Ans: Saturday

Question 45

A circular track has a radius of 6 meters. Yuri runs at a constate rate of 2π meters per second and Zoe runs at a constant rate of 4π meters per second. They start at the same location on the track and begin running in opposite directions. How far will they be from the starting location when they meet for the first time?

Ans: $6\sqrt{3}$

Question 46

Lil' Ptolemy can rap 100 words per minute when he is awake and can somehow rap 50 words per minute when he is asleep. Assuming that Lil' Ptolemy raps for 5 hours while asleep and 10 hours while awake each day, how many words does he rap per day?

Ans: $75,000$

Question 47

Alex plays a video game for 10 hours every day, except on Sundays, on which she doesn't play any video games. Given that she starts playing on a Monday, on which day of the week will she reach her $500th$ hour?

Ans: Tuesday

Question 48

Sam chooses a positive integer that leaves a remainder of 3 when divided by 4. What is the probability that the number he chose is divisible by 7?

Ans: $\frac{1}{7}$

Question 49

Katie took a practice test and originally scored 1500 points. For every three hours she spends studying, she raises her scores on practice tests by 10 points. How many hours does she need to study if she wants to get 1580 points?

Ans: 24 hours

Question 50

Min is playing with a biased coin, which lands heads 70% of time and tails 30% of the time. What is the probability that when Min tosses the coin twice, she gets exactly one head?

Ans: $\frac{21}{50}$