Iowa City Math Circle Handouts
Word Problems

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1 Introduction

Word problems are a general class of math contest problems that require you to translate from a scenario expressed (in words) into a problem to a set of algebraic equations which you can solve quickly. The key to setting up word problems is to choose your variable(s) properly in order to obtain equations that are simple to solve. However, in most cases, choosing the variables is rather straightforward and so is the algebra. That's why it is important you are extremely quick at solving word problems, which aren't hard but should be solved quickly so that you can spend time on the harder questions that test your problem solving skills. In this chapter, we go over several categories of word problems that are common, and provide you with a handful of exercises that you can practice your quickness on.

2 Percentage-related Word Problems

A good amount of word problems involve percentages. Lets take a look at a simple example.

Example 2.1. If x is 40 percent larger than y, and if z is 30 percent smaller than y, by what percent is z smaller than x?

Solution. For these problems, the easiest way to solve them is to set one of the variables to a non-zero constant. This number can be anything because the ratios between the variables won't change. To make calculations easier, let's set $y = 100$ (100 works well for percent problems because an integer percent of 100 will always be an integer). From the problem, this implies $x = 140$ and $z = 70$. Since z is half of x, z is $\boxed{50\%}$ smaller than x. $\triangle$

With these kinds of problems, you can just set some variables to convenient values and solve for the rest to solve them. However, make sure you don't fix the values of too many variables, as that could cause some of the equations or constraints to be violated. A lot of percentage problems also involve money, which could be in the form of taxes, tips, sales, or interest. Let's take a look at a few such problems.

Example 2.2. A person buys a TI-84 calculator for a total of \$86.24. Given that the calculator was bought at a 20% discount and that a 10 percent tax was applied, what was the original price of the calculator?

Solution. A good strategy on this problem is to work backwards. We know the final price the person paid, and we need to find the original price. Lets first find the price of the calculator before the 10% tax. Since the final price is 10 percent higher than the price before tax, we can divide 86.24 by $1 + 10\% = 1.10$ to obtain the price before tax, which is 78.4. To see why we divide by 1.10, let the price before tax be x. Then we have $x + x \cdot 10\% = 86.24$, or $1.1x = 86.24$. Now, we can similarly find the price before the discount. Dividing 78.4 by 0.8(since the discount price is $100 - 20 = 80\%$ of normal price) results in 98 dollars, which is our final answer. △

Lets now take a look at an interest problem. These problems are very similar to population problems because of their shared exponential nature.

Example 2.3. Marisa deposits 2000 dollars into her back account, and it grows at a rate of 3% every year (thanks to interest). How many years would it take for her account to reach \$2600 dollars?

Solution. For this problem we present two solutions. The first is simply calculating the value of the account at the end of every year by multiplying the value of the account 1 year ago by 1.03. To see why the value multiplies by 1.03 every year, let the current value of the account be x. Then, at the end of the year, Marissa gets a 3% boost in value. In terms of x, the value of her account after a year is $x + x \cdot 3\% = 1.03x$. With this method, we find that we would need to multiple 2000 by 1.03 nine times to get a value over 2600, so it would take $\boxed{9}$ years.

Another way to solve this problem is by writing an equation and solving for a variable. We can write this as $2000 \cdot 1.03^t = 2600$, where t is the time in years it takes for her account to reach exactly \$2600. We can simplify this to $1.03^t = 1.3$. Taking the log base 1.03 of each side, we have $t = \log_{1.03} 1.3 \approx 8.88$ (using the fact that $\log_{1.03}(1.03^t) = t \cdot \log_{1.03} 1.03 = t$). We must round this value up as the account grows only at the end of the year, which means it would take 9 years. △

These types of problems appear often, especially on the MATHCOUNTS Target Round, but they are easy to solve once you can understand them.

Checkpoint 2.1. Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of 0, and the score increases by 1 for each like vote and decreases by 1 for each dislike vote. At one point Sangho saw that his video had a score of 90, and that 65% of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point? *Source: AMC*

Checkpoint 2.2. Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was A dollars. The cost of his movie ticket was 20% of the difference between A and the cost of his soda, while the cost of his soda was 5% of the difference between A and the cost of his movie ticket. To the nearest whole percent, what fraction of A did Roger pay for his movie ticket and soda? *Source: AMC*

3 Mixture Problems

Another common category of world problems are mixture problems, where different concentrations of liquids are mixed in, and requires you to have a knowledge on how mixtures work.

Example 3.1. Ellie has a pitcher with 8 ounces of 50% lemonade. How many ounces of 25% lemonade should she add to the pitcher to obtain 35% lemonade? (In general, an $n\%$ drink refers to one in which $n\%$ of its volume is concentrate, and the rest is water).

Solution. We present two methods of solving this problem.
Let us denote the amount of 25% lemonade we must add as x. Our 8 ounce drink consists of $.50 \cdot 8 = 4$ ounces of concentrate. Now, by adding x ounces of 25% lemonade, we will add $.25x$ ounces of concentrate to the pitcher. We are now ready to set up an equation. The pitcher will have a total of $4 + .25x$ ounces of concentrate, and $8 + x$ ounces of lemonade. Therefore, our equation is

$$\frac{4 + .25x}{8 + x} = 35\% = .35.$$

Solving, we obtain $x = \boxed{12}$ ounces.

Now, we take a look at a second approach, using the idea of weighted averages. In our final 35% mixture, let p represent the fraction of this lemonade that is the original 50% lemonade, so $1 - p$ represents the fraction of the final lemonade that is the 25% mixture. Then, we have

$$.50p + .25(1 - p) = .35.$$

Solving, we obtain $p = \frac{2}{5}$, indicating that $\frac{2}{5}$ of our final mixture is the original 8 ounce drink. Therefore, the other $\frac{3}{5}$ is the 25% lemonade. Let x be the amount of 25% lemonade we use. Then we have

$$\frac{x}{8} = \frac{\frac{3}{5}}{\frac{2}{5}},$$

giving us that $x = 12$ ounces, matching our result above. △

Checkpoint 3.1. Ellie's lemonade stand has a pitcher with 4 cups of water. After adding in a saltwater solution to the pitcher, the pitcher now contains 9 cups of 40% saltwater. What is the concentration of the saltwater solution she added?

4 Rate Problems

Another type of word problems that are encountered often are work-related problems , and We will look at two types of them below. In the first type, the workers work at identical rates. Consider the following problem:

Example 4.1. In a specific company, there are lumberjacks that need to cut trees. Assume that all lumberjacks cut down trees at the same rate, and there is no interference between lumberjacks. One day, 20 lumberjacks cut down 100 trees in 60 minutes. At this same rate, how long in minutes would it take for 50 lumberjacks to cut down 50 trees?

In order to figure out how long the job would take, lets take a look at how changes in each of the variables (number of workers and how large the job is) affect this. When the amount of work changes, the amount of time it takes to do the job changes in a directly proportional manner (for example, doubling the size of the job would double the total time needed to complete the job). On the other hand, the number of workers is inversely proportional to the time needed (for example, if we have half as many workers, the time needed to do the job doubles).

Solution. In this problem, the amount of workers is multiplied by $\frac{50}{20} = \frac{5}{2}$, and the size of the job is halved. This means the job time will be multiplied by $\frac{1}{\left(\frac{5}{2}\right)} = \frac{2}{5}$ because of the change in workers, and will be halved by the change in the size of the job. Therefore, given that our original time is 60 minutes, our desired time is $60 \cdot \frac{2}{5} \cdot \frac{1}{2} = \boxed{12}$ minutes. △

For these problems, a general theorem can be used. Let t_1, s_1, and w_1 represent the time of the job, size of the job, and workers respectively. We then can use t_2, s_2, and w_2 in a similar way. Then,

$$\frac{w_1 \cdot t_1}{s_1} = \frac{w_2 \cdot t_2}{s_2}.$$

This can be also used to solve the previous example.

The second type of work problem involves multiple workers that work at different rates. Consider the following example.

Example 4.2. Reimu and Marisa are mowing the lawn, and assuming they work at constant rates. If Reimu were to mow the lawn alone, it would take her 20 minutes for her to complete the job. If both were working together mowing the lawn, then it would them 15 minutes to finish, assuming both are working without interference. How long in minutes would it take for Marisa to mow the lawn if she works alone?

Solution. In order to crack this problem, we need to first find the rates of how fast Reimu works and how fast both work together. Since Reimu takes 20 minutes to mow the lawn, she completes $\frac{1}{20}$th of the job in one minute. Similarly, both would complete $\frac{1}{15}$th of the job in one minute. We can now calculate how much Marisa completes the job in one minute, by subtracting how much Reimu does from how much the team does in one minute. $\frac{1}{15} - \frac{1}{20} = \frac{1}{60}$, so Marisa completes $\frac{1}{60}$th of the job in one minute. Therefore, it would take her $\boxed{60}$ minutes to complete the job. △

In order to solve these problems, we need to convert what is given in problem into how much work a person (or persons) does in a unit of time. After that, we can calculate how much work the desired unit(s) will do in that unit of time, and with that we can calculate how long the unit(s) will take to complete the job. In contests, these types of problems can also be in the form of hoses filling up a pool.

5 Distance-Speed-Time Problems

The two following formulas are commonly used in Distance-Speed-Time problems:

1. $d = st$, $s = \frac{d}{t}$, $t = \frac{d}{s}$
2. $$s_{\text{avg}} = \frac{d_{\text{total}}}{t_{\text{total}}} = \frac{d_1 + d_2 + \ldots + d_k}{t_1 + t_2 + \ldots + t_k} = \frac{d_1 + d_2 + \ldots + d_k}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \ldots + \frac{d_k}{s_k}}$$
   for k legs of travel.

Example 5.1. Kylie took a road trip in which she drove at 60 mph for the first 90 minutes, 40 mph for the next 60 miles, and covered the last 100 miles of her trip in 50 minutes. What was her average speed for the whole trip?

Solution. An easy mistake to do is to simply calculate the average of all the speeds mentioned in that problem. However, we can not do that because the time spent at the different speeds are different, so they can not be weighted equally.

To find Kylie's average speed, we must calculate the total distance she travelled, along with the total time of travel. To find these quantities, we will calculate the distance and time for each of the three legs of travel.

For the first leg, we are given that she drove at 60 mph for 90 minutes. To find the distance travelled, we use $d = st$. Noting that 90 minutes is $\frac{90}{60} = \frac{3}{2}$ hours, we have the distance travelled is $d = (60)(\frac{3}{2}) = 90$ miles. The time spent on this leg is given to us - it is simply 90 minutes, or $\frac{3}{2}$ hours. (Make sure you keep your units consistent!)

Now, for the second leg, we are given how far she travels: 60 miles. To calculate the time spent on this leg, we use the formula $t = \frac{d}{s} = \frac{60}{40} = \frac{3}{2}$ hours.

For the final leg, we are given both the distance and time travelled! Specifically, she covers 100 miles in $\frac{50}{60} = \frac{5}{6}$ hours.
Using our values of d_1, d_2, d_3, t_1, t_2, and t_3 we calculated above, we are ready to get the numerical answer to our problem:

$$s_{\text{avg}} = \frac{90 + 60 + 100}{\frac{3}{2} + \frac{2}{3} + \frac{5}{6}} = \frac{250}{3}$$

Hence, Kylie's average speed is $\boxed{\dfrac{250}{3}}$ mph.

Checkpoint 5.1. Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph, and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes? *Source: AMC*

$\triangle$

Another frequently-appearing scenario involves two people who are moving simultaneously at different speeds. Given this sort of scenario, you are often tasked with finding when and/or where they meet. There is a nifty trick to solve this, as discussed in the following example.

Example 5.2. Kevin and Reece live in houses 90 miles apart (the distance between Cedar Falls and Iowa City). They both leave their houses and drive to the other person's house along a straight line. Kevin is driving at a constant speed of 75 mph(he tends to speed a little!), whereas Reece drives at a constant speed of 60 mph. How long after they start driving do they cross paths? Additionally, how far is the point at which they meet from each house?

Solution. Each minute after they both leave, Reece travels 1 mile and Kevin travels $\frac{75}{60} = 1.25$ miles. Therefore, the distance separating them decreases by $1 + 1.25 = 2.25$ miles every minute. This means that they meet in $\frac{90}{2.25} = \boxed{40}$ minutes. In 40 minutes, Reece travels $\boxed{40}$ miles whereas Kevin travels $75 \cdot \frac{40}{60} = \boxed{50}$ miles from their respective houses. Note that we can check our answers by seeing that $40+50 = 90$ (i.e. the sum of the distance traveled by each person equals the initial distance between the two). $\triangle$

Next, we look at a problem which may seem tedious to solve, but can actually be solved quickly after a simple observation.

Example 5.3. A car drives from town A to B, a distance of 100 miles, at a speed of 60 miles per hour. At the same time the car leaves A, a fly leaves B in the direction of A, moving at a speed of 30 miles per hour. Upon meeting the car, the fly turns back around, and flies back to B. Once it reaches B, it turns around (in the direction of A), and repeats the process. Once the car reaches B, the fly stops moving. How much distance has the fly covered in total?

Solution. You may be tempted to calculate how much the fly travels in each step (which would involve calculating each time the fly and the car meet up, as well as whenever the fly reaches B). However, this becomes quite tedious. We can exploit the fact that the fly travels at a constant speed of 30 mph throughout the entire journey. Therefore, to calculate the distance the fly traveled, it suffices to find the time the fly travels. This is easy - we know that the fly travels in the same time frame that the car travels. Given the speed of the car and the distance between A and B, we calculate that the car drives for $t = \frac{d}{s} = \frac{100}{60} = \frac{5}{3}$ hours. Therefore, we can now deduce that the fly covers $d = st = 30 \cdot \frac{5}{3} = \boxed{50}$ miles. $\triangle$

As illustrated in the examples, the key to solving distance-speed-time problems is set up the $d = st$ equation for each of the people/objects that are moving in the problem. The exercises at the end will help you practice this and also help you practice setting up simple equations in other areas of word problems, which can be quite tricky.

6 Statistics and Data Analysis

- The *mean*, or *average*, of a collection of values is the sum of the values divided by the number of values: the mean of the data collection $x_1, x_2, \cdots, x_n$ is $(x_1 + x_2 + \cdots + x_n)/n$. For example, the mean of the set $\{1, 3, 5, 6\}$ would be $(1+3+5+6)/4 = 3.75$.

- The *median* of a collection of numbers is the value that evenly separates the data into higher and lower values; in other words, the median is the number in the "middle" of the data when its sorted in ascending order. For example, the median of $2, 5, 8, 10, 11$ is 8. (When finding the median, make sure you place the numbers in *increasing* order first!) If there are an even number of values, the median is the average of the two middle values; for example, the median of $1, 4, 5, 7$ would be $(4 + 5)/2 = 4.5$.

- The *mode* is the value that occurs the most often in a data set. For example, 3 is the unique mode of the set $\{1, 2, 3, 3, 3, 4\}$. There can be multiple modes in a data set if the most frequent values occur with the same frequency. If a data set has only one mode, it is referred to as an *unique mode*.

- The *range* of a data set is the difference between the greatest and least values in the set. For example, the range of the set $\{1, 5, 9, 12\}$ is $12 - 1 = 11$.

Example 6.1. What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17$, and x is equal to the mean of those five numbers? *Source: AMC*

Solution. The mean of the five numbers will be $\frac{4+6+8+17+x}{5} = \frac{35+x}{5}$. There are three possibilities for the median of the numbers:

1. If $x \le 6$, then 6 is the median. In this case, $\frac{35+x}{5} = 6 \implies x = -5$. $-5 \le 6$ so this is a valid value for x.

2. If $6 < x < 8$, then x is the median. In this case, $\frac{35+x}{5} = x \implies x = 8.75$. However, $8.75 > 8$, so this value for x does not work.

3. If $x \ge 8$, then 8 is the median. In this case, $\frac{35+x}{5} = 8 \implies x = 5$. However, $5 < 8$, so this value for x also does not work.

Overall, $x = -5$ is the only possible value that satisfies the condition, so our answer is $\boxed{-5}$. △

Checkpoint 6.1. The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x? *Source: AMC*

6.1 Reading Graphs and Data Plots

In a stem and leaf plot, the tens digit of all the data values are placed in the left column, and the corresponding ones digits are on the right side.

Example 6.2. (Stem and Leaf Plots)

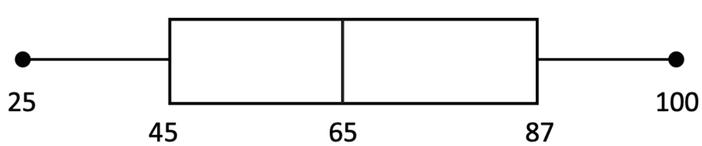
2	3 4 6 7
3	3 8 8 8 8
4	1 1 5

The stem-and-leaf plot shows the number of points scored by the winning team in each of the first 12 NFL games played this football season. What is the absolute difference between the mode and median of this data? *Source: MATHCOUNTS*

Solution. The most frequently occurring value is 38. The median is also 38. So the absolute difference between the mode and median is $\boxed{0}$. △

The *first quartile* is the "middle number" between the least number and the median of a collection of values, and the *third quartile* is the "middle number" between the median and the greatest value (the *second quartile* would be the median). The *interquartile range* is the difference between the third quartile and the first quartile.

Example 6.3. (Box and Whisker Plots)

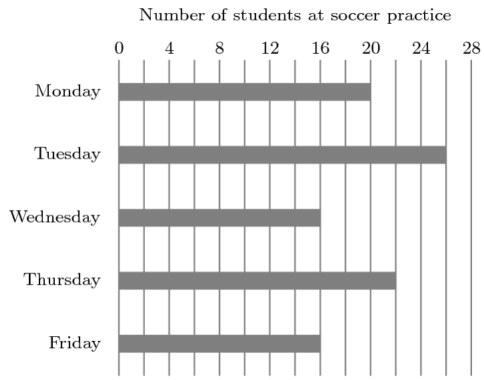


What is the positive difference between the range and the interquartile range of the data set represented by this box-and-whisker plot? *Source: MATHCOUNTS*

Solution. The range of the data set is $100 - 25 = 75$, and the interquartile range is $87 - 45 = 42$. The positive difference between the two is $\boxed{33}$.
Alternatively, the positive difference between the range and the interquartile range can also be computed as $(45 - 25) + (100 - 87) = 33$. △

6.2 Review Exercises

1. ⋆ The diagram shows the number of students at soccer practice each weekday during last week. After computing the mean and median values, Coach discovers that there were actually 21 participants on Wednesday. Which of the following statements describes the change in the mean and median after the correction is made?



(A) The mean increases by 1 and the median does not change.

(B) The mean increases by 1 and the median increases by 1.

(C) The mean increases by 1 and the median increases by 5.

(D) The mean increases by 5 and the median increases by 1.

(E) The mean increases by 5 and the median increases by 5.

Source: AMC 8

2. $\star$ Laila took five math tests, each worth a maximum of 100 points. Laila's score on each test was an integer between 0 and 100, inclusive. Laila received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Laila's score on the last test? *Source: AMC 8*

3. $\star\star$ Melanie computes the mean μ, the median M, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consists of 12 1s, 12 2s, $\ldots$, 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

   (A) $\mu < d < M$ **(B)** $M < d < \mu$ **(C)** $d = M = \mu$ **(D)** $d < M < \mu$ **(E)** $d < \mu < M$

   Source: AMC

4. $\star\star$ A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list? *Source: AMC*

5. $\star\star$ When the mean, median, and mode of the list

$$10, 2, 5, 2, 4, 2, x$$

   are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x? *Source: AMC*

6. $\star\star$ The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. Find the largest integer that can be an element of this collection. *Source: AMC*

7 Exercises

1. $\star$ Ellie has a saltwater solution. If 5 ounces of water were to evaporate from her solution, her new solution would be 80% saltwater. If she were to add 4 ounces of water to her original solution, her solution would be 50% saltwater. Find the concentration and volume of her original solution.

2. $\star$ Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk? *Source: AMC*

3. $\star$ For every dollar Ben spent on bagels, David spent 25 cents less. Ben paid \$12.50 more than David. How much did they spend in the bagel store together? *Source: AMC*

4. $\star$ At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets? *Source: AMC*

5. $\star$ The student council sold 661 T-shirts, some at \$10 and some at \$12. When recording the number of T-shirts they had sold at each of the two prices, they reversed the amounts. They thought they made \$378 more than they really did. How many T-shirts actually were sold at \$10 per shirt? *Source: MATHCOUNTS*

6. $\star$ A car travels 40 kph for 20 kilometers, 50 kph for 25 kilometers, 60 kph for 45 minutes and 48 kph for 15 minutes. What is the average speed of the car, in kph? *Source: MATHCOUNTS*

7. $\star$ Anna ran to her friend's house at a rate of 8 miles per hour. On the way back, she ran the same route in reverse, but she ran at a rate of 6 miles per hour. If Anna's route is one mile long each way, how many minutes longer did it take her to run back from her friend's house than it took her to run to her friend's house? *Source: MATHCOUNTS*

8. $\star$ Rico can walk 3 miles in the same amount of time that Donna can walk 2 miles. Rico walks a rate 2 miles per hour faster than Donna. At that rate, what is the number of miles that Rico walks in 2 hours and 10 minutes? *Source: MATHCOUNTS*

9. $\star$ Andrew can paint a room in 3 hours. Bob can paint the same room in 6 hours. How many hours will it take Andrew and Bob to paint the room together? *Source: AoPS*

10. $\star$ Millie drives from home to the Fixit Garage at a rate of 30 mph. She leaves her car and rides a bus back home. The bus makes the trip at a rate of 20 mph. If her total travel time for the round trip is one hour, how many miles from the home is the Fixit Garage? *Source: MATHCOUNTS*

11. $\star$ Qiang drives 15 miles at an average speed of 30 miles per hour. How many additional miles will he have to drive at 55 miles per hour to average 50 miles per hour for the entire trip? *Source: AMC 8*

12. $\star$ A store increased the original price of a shirt by a certain percent and then decreased the new price by the same amount. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased? *Source: AMC 8*

13. $\star$ On a trip to the beach, Anh traveled 50 miles on the highway and 10 miles on a coastal access road. He drove three times as fast on the highway as on the coastal road. If Anh spent 30 minutes driving on the coastal road, how many minutes did his entire trip take? *Source: AMC 8*

14. $\star$ Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is $10,560$ feet, and Bella covers $2\frac{1}{2}$ feet with each step. How many steps will Bella take by the time she meets Ella? *Source: AMC 8*

15. $\star$ Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers? *Source: AMC 8*

16. $\star\star$ David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home? *Source: AMC*

17. $\star\star$ Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips? *Source: AMC 8*

18. $\star\star$ Mr. Earl E. Bird gets up every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives

at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time? *Source: AMC*

19. $\star\star$ Minnie rides on a flat road at 20 kilometers per hour (kph), downhill at 30 kph, and uphill at 5 kph. Penny rides on a flat road at 30 kph, downhill at 40 kph, and uphill at 10 kph. Minnie goes from town A to town B, a distance of 10 km all uphill, then from town B to town C, a distance of 15 km all downhill, and then back to town A, a distance of 20 km on the flat. Penny goes the other way around using the same route. How many more minutes does it take Minnie to complete the 45-km ride than it takes Penny? *Source: AMC*

20. $\star\star$ Elmer's new car gives 50% percent better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is 20% more expensive per liter than the gasoline his old car used. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip? *Source: AMC*

21. $\star\star$ Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden? *Source: AMC*

22. $\star\star$ Two hoses, A and B, are used to fill a fish tank with water. Hose A puts water into the tank twice as fast as hose B. If both hoses are used, the tank is filled five minutes faster than if just hose A is used. How many minutes would it take for hose B to fill the tank on its own? *Source: AoPS*

23. $\star\star$ Abe can paint the room in 15 hours, Bea can paint 50 percent faster than Abe, and Coe can paint twice as fast as Abe. Abe begins to paint the room and works alone for the first hour and a half. Then Bea joins Abe, and they work together until half the room is painted. Then Coe joins Abe and Bea, and they work together until the entire room is painted. Find the number of minutes after Abe begins for the three of them to finish painting the room. *Source: AIME*

24. $\star\star\star$ Jon and Steve ride their bicycles along a path that parallels two side-by-side train tracks running the east/west direction. Jon rides east at 20 miles per hour, and Steve rides west at 20 miles per hour. Two trains of equal length, traveling in opposite directions at constant but different speeds each pass the two riders. Each train takes exactly 1 minute to go past Jon. The westbound train takes 10 times as long as the eastbound train to go past Steve. Find the length of each train in miles. *Source: AIME*